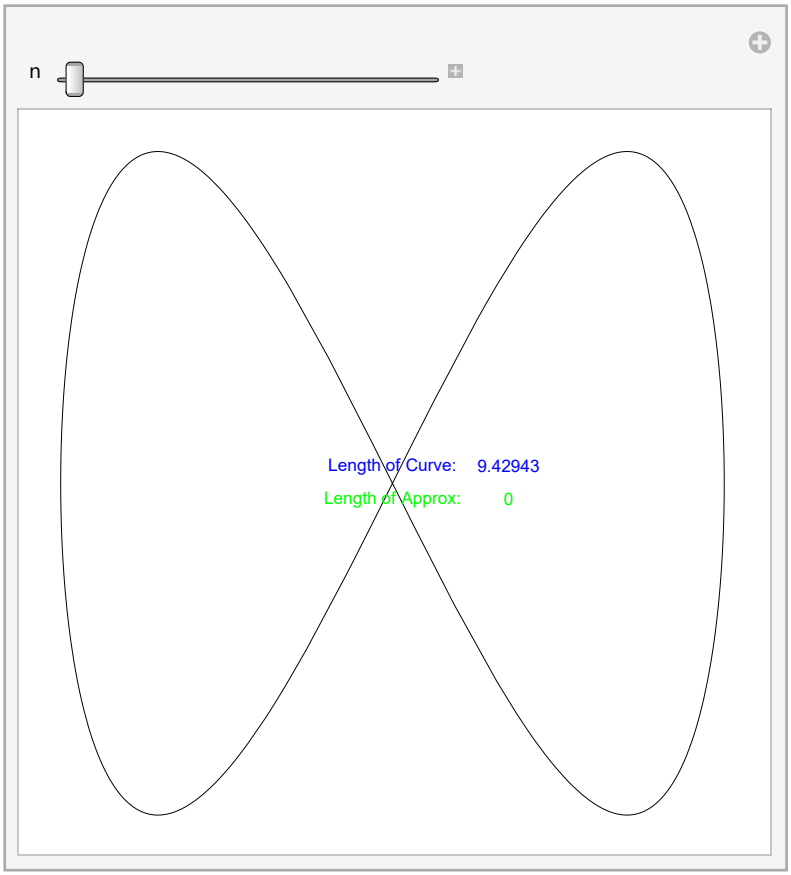
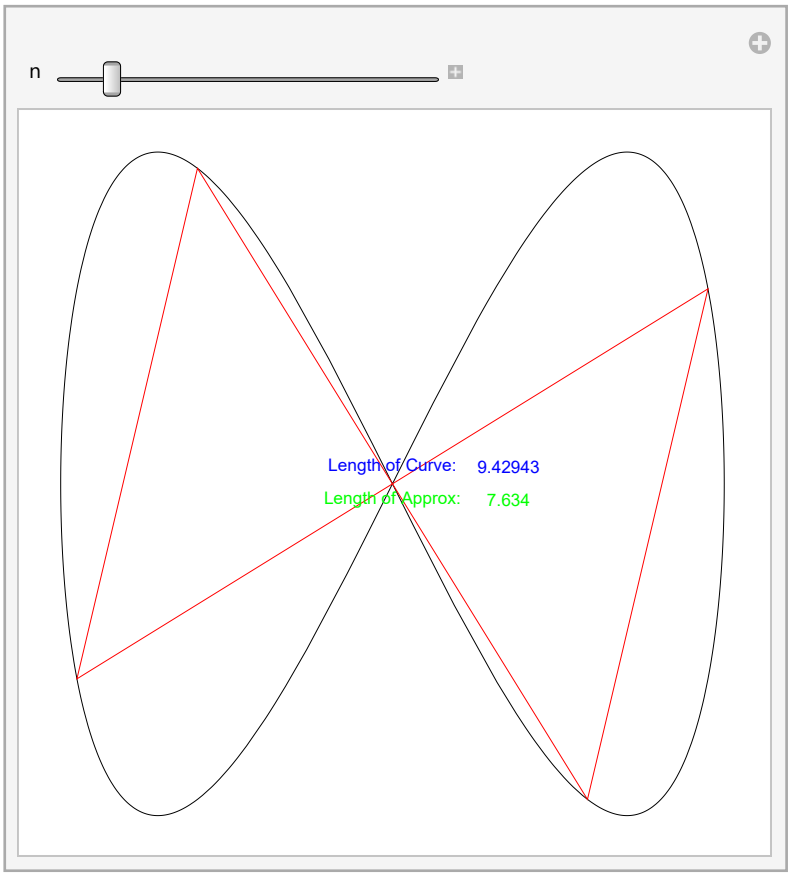
```
Func[f_, range_] := Manipulate[
  n = Floor[n];
  approx =
   Table[f /. range[[1]] → j, {j, range[[2]], range[[3]], (range[[3]] - range[[2]])/n}];
  Graphics[
   {ParametricPlot[f, range][[1]][[1]][[3]][[1]][[2]],
    Red, Line[approx],
    Blue, Text["Length of Curve:", {0, 0.05}],
    Text[N[NIntegrate[Norm[D[f, range[[1]]]], range], 3], {0.35, 0.05}],
    Green, Text["Length of Approx:", {0, -0.05}],
    Text[N[Plus @@ Function[Norm[{#1, #2}]] @@@ Differences[approx], 4], {0.35, -0.05}]
   }], {n, 2, 30}]
Func[{Sin[t], Sin[2 t]}, {t, 0, 2 Pi}]
```



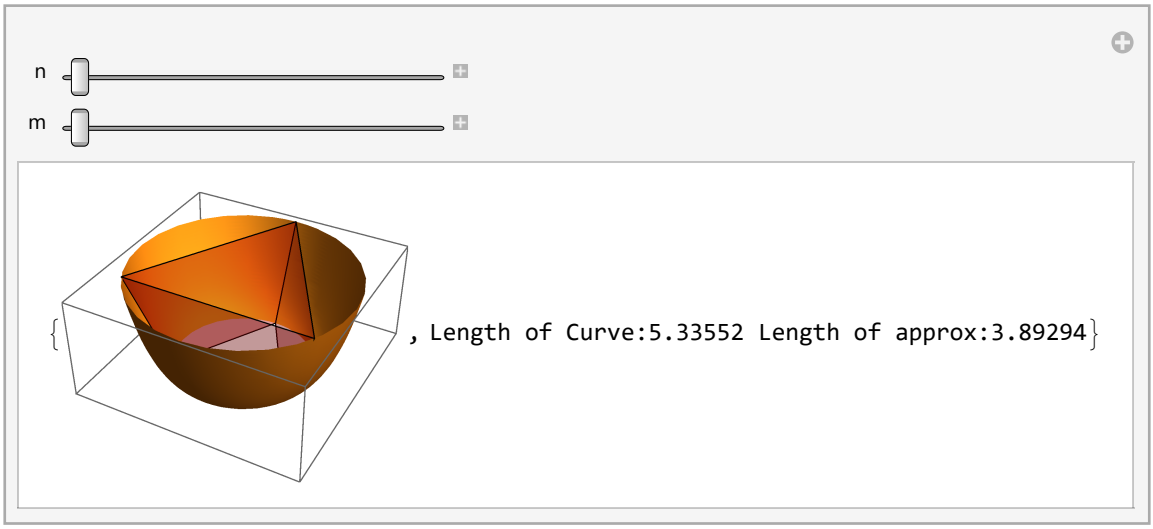
```
Func3D[f_, rangex_, rangey_] := Manipulate[
  n = Floor[n]; m = Floor[m];
  xvar = rangex[[1]];
  yvar = rangey[[1]];
  dx = (rangex[[3]] - rangex[[2]])/n;
  dy = (rangey[[3]] - rangey[[2]])/m;
  approx = Flatten[Table[Polygon[{f /. {xvar → j, yvar → k}, f /. {xvar → j + dx, yvar → k},
       f /. {xvar → j + dx, yvar → k + dy}, f /. {xvar → j, yvar → k + dy}}],
     {j, rangex[[2]], rangex[[3]] - dx, dx}, {k, rangey[[2]], rangey[[3]] - dy, dy}], 1];
  {Graphics3D[
    {ParametricPlot3D[f, rangex, rangey, Mesh → 0][[1]],
     Red, Opacity[0.4], approx,
     (*Blue,Text["Length of Curve:",{0,0,0}],
     Text[Integrate[Norm[Cross[D[f,xvar],D[f,yvar]]],rangex,rangey]],
     Green,Text["Length of Approx:",{0,0,0}], Text[N@Plus@@Map[Area,approx],{0,0,0}]*)
    }]
   , "Length of Curve:" <>
    ToString[N[NIntegrate[Norm[Cross[D[f, xvar], D[f, yvar]]], rangex, rangey], 3]] <>
    " Length of approx:" <> ToString[N[Plus @@ Map[Area, approx], 3]]}
  , {n, 3, 30}, {m, 1, 30}]
```

```
Func[{Sin[t], Sin[2 t]}, {t, 0, 2 Pi}]
```



In this 2D case the length of the curve is sup of the linear approximation

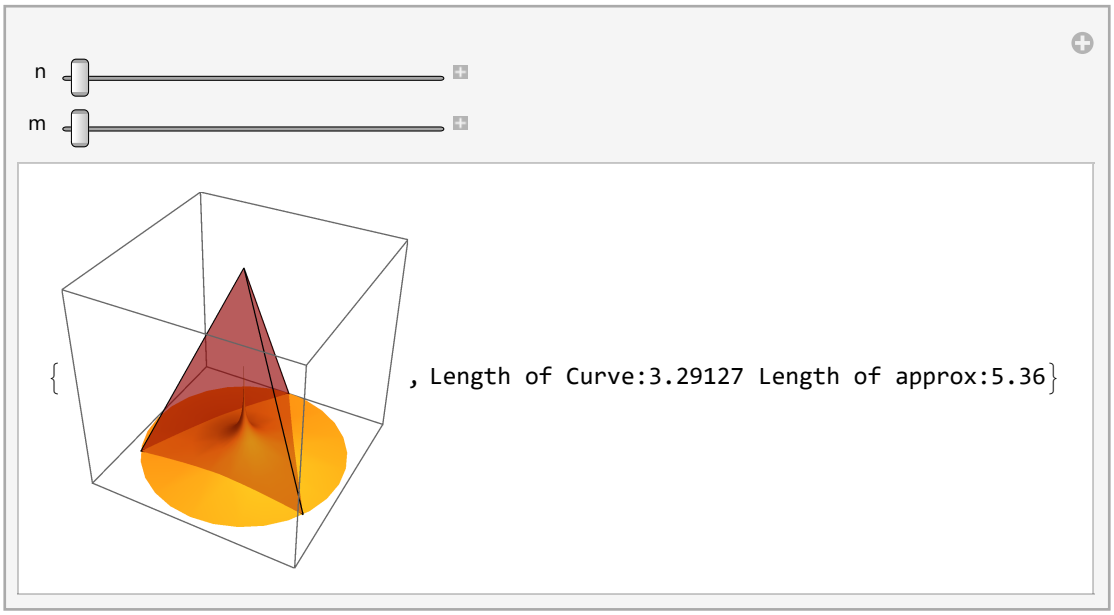
```
Func3D[{Sin[t] * u^0.2, Cos[t] * u^0.2, u}, {t, 0, 2 Pi}, {u, 0.1, 1}]
```



... General: Further output of Area::reg will be suppressed during this calculation.

In this 3D case the area of the surface is still sup of the linear approximation

```
Func3D[{Sin[t] * u^-10, Cos[t] * u^-10, u}, {t, 0, 2 Pi}, {u, 1, 3}]
```



In this case the area of the linear approximation is greater than the area of the surface so the area is not the sup of the linear approximation